Activity 16

Signals and Systems

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Name____

1)Inverse Fourier Transform

Determine the inverse Fourier transforms of the following:

a) $F(\omega) = 2\pi[\delta(\omega-4) + \delta(\omega+4)] + 8\pi\delta(\omega)$.

b) $A(\omega) = 6\pi\cos(5\omega)$.

c) $B(\omega) = \sum_{n=-\infty}^{\infty} 2\pi \frac{1}{1+n^2}\delta(\omega - n2)$.

d) $C(\omega) = \frac{8}{j\omega} + 4\pi\delta(\omega)$.

a)

$$f(t) = 2\cos(4t) + 4.$$

b)

Using Euler's identity,

$$A(\omega) = 3\pi e^{j\omega 5} + 3\pi e^{-j\omega 5}.$$

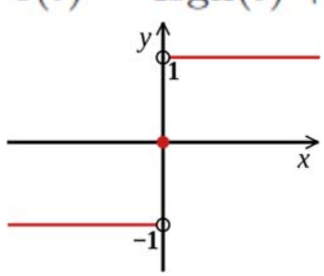
$$a(t) = 3\pi\delta(t+5) + 3\pi\delta(t-5).$$

c)

$$b(t) = \sum_{n=-\infty}^{\infty} \frac{1}{1+n^2} e^{j2nt}.$$

d)

$$c(t) = 4\text{sgn}(t) + 2.$$



sgn(x) function (only as reference)

2) Nyquist Theorem Concept

If signal $f(t)$ is not band-limited, would it be possible to reconstruct $f(t)$ exactly from its samples $f(nT)$ taken with some finite sampling interval $T > 0$? Explain your reasoning.

The answer is no. According to the Nyquist criterion, we can have perfect reconstruction, only if our sampling interval T fulfill the following relation

$$T < \frac{1}{2B},$$

where B is the signal bandwidth. Then, if the signal is not band-limited, this ratio approaches zero. Therefore, any $T > 0$ will not satisfy the Nyquist criterion, and will cause aliasing error.

3) Nyquist Theorem Application

The inverse of the sampling interval T–that is, T^{-1}–, is known as the sampling frequency and usually is specified in units of Hz. Determine the minimum sampling frequencies T^{-1} needed to sample the following analog signals without causing aliasing error:

a) Arbitrary signal $f(t)$ with bandwidth 20 kHz.

b) $f(t) = \text{sinc}(4000\pi t)$.

c) $f(t) = \text{sinc}(4000\pi t)\cos(20000\pi t)$.

a) Using the Nyquist criterion, we have

$$T < \frac{1}{2B} = \frac{1}{40\text{kHz}}.$$

Therefore, the minimum sampling frequency is

$$T_{\min}^{-1} = 40\text{kHz}.$$

b) Since,

$$f(t) = \text{sinc}(4000\pi t) \longleftrightarrow \frac{1}{4000}\text{rect}\left(\frac{\omega}{8000\pi}\right)$$

then the bandwidth of $f(t)$is $B = 4000\pi\text{rad/s} = 2000\text{Hz}$. Next, using the Nyquist criterion, we have

$$T < \frac{1}{2B} = \frac{1}{4\text{kHz}}.$$

Therefore, the minimum sampling frequency is

$$T_{\min}^{-1} = 4\text{kHz}.$$

c) In this case, we have the signal from part (b), but modulated by $\omega_c = 20000\pi\text{rad/s}$. Consequently, the bandwidth of the signal $f(t)$is

$$B = 20000\pi + 4000\pi\text{rad/s} = 24000\pi\text{rad/s} = 12\text{kHz}.$$

Hence,

$$T_{\min}^{-1} = 2B = 24\text{kHz}.$$

4) Nyquist Theorem Sampling

Using

$$\sum_{n=-\infty}^{\infty} f(t)\delta(t-nT) \leftrightarrow \sum_{n=-\infty}^{\infty} \frac{1}{T}F(\omega - n\frac{2\pi}{T})$$

sketch the Fourier transform $F_T(\omega)$ of signal

$$f_T(t) \equiv \sum_{n=-\infty}^{\infty} f(t)\delta(t-nT)$$

if

$$f(t) = \cos(4\pi t),$$

assuming (a) $T = 1$ s, and (b) $T = 0.2$ s. Which sampling period allows $f(t)$ to be recovered by applying an appropriate low-pass filter to $f_T(t)$?

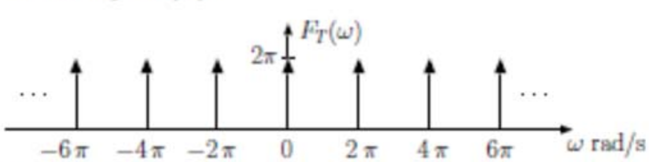
a) With $T = 1\text{s}$,

$$F_T(\omega) = \sum_{n=-\infty}^{\infty} F(\omega - n2\pi) = \pi \sum_{n=-\infty}^{\infty} \left[\delta(\omega - 4\pi - 2n\pi) + \delta(\omega + 4\pi - 2n\pi)\right].$$

We notice that we can rewrite this expression as:

$$F_T(\omega) = 2\pi \sum_{n=-\infty}^{\infty} \delta(\omega - 2n\pi)$$

Sketching $F_T(\omega)$:

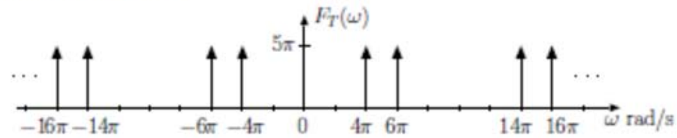


Notice, that in this case, it is impossible to recover $f(t)$ from $F_T(\omega)$ with a low-pass filter, because $F_T(\omega)$ contains aliased components at 0 and $\pm 2\pi$.

b) With $T = 0.2$s,

$$F_T(\omega) = \sum_{n=-\infty}^{\infty} 5F(\omega - n10\pi) = 5\pi \sum_{n=-\infty}^{\infty} \left[\delta(\omega - 4\pi - 10n\pi) + \delta(\omega + 4\pi - 10n\pi)\right].$$

Sketching $F_T(\omega)$:



In case (b), the original $f(t)$ can be recovered from $F_T(\omega)$ by applying an appropriate LP filter.