Activity 17

Signals and Systems

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1. An LTI circuit has the frequency response $H(\omega) = \frac{1}{1+j\omega} + \frac{1}{2+j\omega}$. What is the system impulse response $h(t)$ and what is the system response $y(t) = h(t) * f(t)$ to the input $f(t) = e^{-t}u(t)$?

   Solution:
   We can obtain the system impulse response $h(t)$ by taking the inverse Fourier transform of $H(\omega)$,

   $$h(t) = e^{-t}u(t) + +e^{-2t}u(t).$$

   To obtain the system response, we first analyze in the frequency domain,

   $$\begin{aligned} Y(\omega) &= H(\omega)F(\omega) = \left(\frac{1}{1+j\omega} + \frac{1}{2+j\omega}\right)\left(\frac{1}{1+j\omega}\right) \\ &= \frac{1}{(1+j\omega)^2} + \frac{1}{1+j\omega} - \frac{1}{2+j\omega}. \end{aligned}$$

   Finally, taking the inverse Fourier transform of $Y(\omega)$, we obtain

   $$y(t) = \left(te^{-t} + e^{-t} - e^{-2t}\right)u(t).$$

2. Find the impulse responses $h(t)$ of the systems having the following frequency responses:

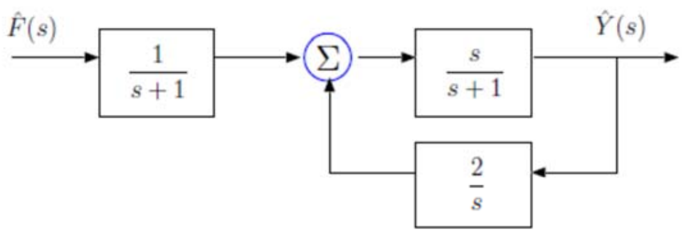
   a) $H(\omega) = \frac{1}{3+j\omega}$.
   b) $H(\omega) = \frac{1}{(4+j\omega)^2}$.
   c) $H(\omega) = \frac{j\omega}{5+j\omega} = 1 - \frac{5}{5+j\omega}$.
   d) $H(\omega) = \frac{1}{1+j\omega}e^{-j\omega}$.

   Solution:

   a) $h(t) = e^{-3t}u(t)$.
   b) $h(t) = te^{-4t}u(t)$.
   c) $h(t) = \mathcal{F}^{-1}\{1\} - 5\mathcal{F}^{-1}\left\{\frac{1}{5+j\omega}\right\} = \delta(t) - 5e^{-5t}u(t)$.
   d) $h(t) = \left[\mathcal{F}^{-1}\left\{\frac{1}{1+j\omega}\right\}\right]\Big|_{t=t-1} = e^{-(t-1)}u(t-1)$.

3)

Determine the transfer function $H(s)$ of the system shown below. Also determine whether the system is BIBO stable.



Solution:

Let the output of the sum be

$$\hat{Q}(s) = \frac{1}{s+1}\hat{F}(s) + \frac{2}{s}\hat{Y}(s).$$

Then the output can be written as

$$\begin{aligned} \hat{Y}(s) &= \hat{Q}(s)\frac{s}{s+1} = \frac{s}{(s+1)^2}\hat{F}(s) + \frac{2}{s+1}\hat{Y}(s) \\ \hat{Y}(s)\left[1 - \frac{2}{s+1}\right] &= \frac{s}{(s+1)^2}\hat{F}(s). \end{aligned}$$

Consequently the transfer function is

$$\hat{H}(s) = \frac{\hat{Y}(s)}{\hat{F}(s)} = \frac{s}{(s+1)(s-1)}.$$

The system is not BIBO stable, because it has one pole $(s = 1)$ in the RHP.